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# REVIEWS

**The Tricomi Equation with Applications to the Theory of Plane Transonic Flow.** By A. R. MANWELL. Pitman, 1979. 185 pp. £8.50.

**Numerical Methods for the Computation of Inviscid Transonic Flows with Shock Waves.** Edited by A. RIZZI & H. VIVIEND. Vieweg, 1981. 266 pp. DM 72-00.

Transonic aerodynamics has had, in its long history, its ups and downs.

From the seeds sown by Molenbroek, Chaplygin and Meyer in their fundamental papers about 1900 describing aspects of compressible flow and introducing the hodograph transformation, and nurtured up through the early 1900s by Taylor, Tricomi and Frankl with further understanding of mixed flows and partial differential equations, the topic blossomed into full vigour across the world during the decade 1947–1957 with classic contributions from Lighthill, Cherry, Germain, Busemann, Guderly, Ferrari, Friedrichs, Von Mises and Morawetz. During this period of creative analytic activity solutions for a range of restricted types of transonic flow were studied, and questions of uniqueness and existence were intensely debated. In particular concern was focused on the mathematical conditions for shock-free supersonic flow regions embedded in subsonic flow.

By the beginning of the 1960s, although fundamental questions remained un answered activity diminished because the limit of analytic methodologies had almost been reached. Moreover the space age had arrived, so far more challenging problems (and money) were to be found elsewhere. Transonic aerodynamics was sustained during this period by lone experimentalists, in particular Pearcey at the NPL in the UK and Whitcome in the USA. It was due to their patient intuitive accumulation of physical insight which brought the gradual realization that by careful aerofoil design the appearance of shock waves could be delayed to higher free-stream Mach numbers, thus improving aircraft performance considerably. The emergence of this realization at the same time that computer technology had become sufficiently developed gave the impetus for a new era of transonics based on numerical methods, supported by careful experimental data. A new generation of inventive and active numerical analysts, Boestel, Jameson, Murman, Garabedian, Balhaus, Steger, have appeared on the scene. So once again transonic aerodynamics is alive and exciting.

The coincidental and simultaneous appearance of these two complementary monographs contrasts nicely the elegance and subtleties of the past analytic era with the contemporary harnessing of brute force numerical power and powerful algorithms. Whereas the account of the past by Manwell is more or less complete the account edited by Rizzi/Viviend is already out of date.

Manwell's book is a concise, succinct, and readable description of the rigorous analytical theory of an inviscid fluid flowing at high subsonic Mach numbers past a restricted range of two-dimensional profiles. After an informative and critical review in non-mathematical terms of the historical development, Manwell describes briefly some standard techniques for solving linear equations of second-order and of mixed type, the equations of plane transonic flow in the physical and hodograph planes, maximum principles and uniqueness theorems, solutions of the Euler–Poisson–Darboux equation, weak shock wave solutions and the main argument in the transonic controversy. This controversy revolves around the work of Morawetz who showed that if there is a smooth flow around a profile (i.e. a mixed subsonic/supersonic flow without shocks) then for arbitrary smooth perturbations of that profile no adjacent smooth flow solutions can be found. But Ferrari and Tricomi disagree because of doubt about the definition of a 'profile'. A consequence of the above theorem is that mathematically a smooth mixed flow past an aerofoil without a shock is unlikely; the fact that modern aerofoils have extensive embedded supersonic regions without shocks is unpalatable to the mathematician, a fact which Manwell fully recognizes and acknowledges. It has been suggested, but not mentioned by Manwell, that the three-dimensionality of the real world could well relax conclusions based on two-dimensionality. Manwell concludes that analytic progress might be feasible by widening the solution of the hodograph equations by considering nonlinear boundary conditions.

The monograph edited by Rizzi and Viviend is totally different. It reports the outcome of a Workshop held in Sweden to compare results of different numerical methods developed across the western world applied to a set of problems of high subsonic inviscid flows past two-dimensional (non-lifting and lifting) aerofoils.

There are three main groups of field solution; numerical solution of the full nonlinear potential equations with conservation of mass across any 'shock' discontinuities (i.e. fully conservative solution); numerical solution of the full nonlinear potential equation without conservation of mass across any 'shock' discontinuities, (i.e. non-conservative solution); numerical solution of the Euler equations in conservative form. Within these groups there are different approaches, either by finite differences, finite volumes, or finite elements, with various levels of fast algorithms to solve the discrete representation of the equations, e.g. relaxation, approximate factorization or multi-grid. It is recognized that the solution of the Euler equations is, in principle, the more exact since 'shock' discontinuities approximate closely to Rankine–Hugoniot shocks, and the correct rotational conditions are obtained aft of the shock. But such programs tend to be long, so many workers choose the easier approach of neglecting entropy changes across a 'shock' discontinuity and assuming potential flow throughout the field. Unfortunately, when the conservative form of solution of the potential equation is obtained, the 'shock' discontinuity is significantly larger and located further aft along the aerofoil chord than a corresponding Rankine–Hugoniot shock, whereas the incorrect non-conservative solution of the potential equation gives 'shock' discontinuities which line up much more closely with the solution of the Euler equations. Non-conservative solutions of these potential equations are not unique; nevertheless such non-conservative solutions play an invaluable role at the present time in the aircraft industries.

This whole area is one of transition in which rapid progress is taking place in the understanding of the ranges of validity of the different types of solution, and in the development of efficient numerical algorithms, possibly interrelating and phasing in with the new generation of computers either of the vector-processing array type or the distributed-array processor type. The general reader will be able to appreciate the trends but the outlines of the various methods given in the book are cursory, so for detailed background the general reader will have to go to the original sources. The specialist will be intrigued by the differences between the different methods in each group of solutions.

Overall, as far as this reviewer is concerned, both of these monographs are welcome additions to his bookshelf. Manwell has put into perspective the mathematical rigour of plane transonic flows whereas Rizzi/Viviend have performed a useful service in clarifying the state of the art of about a year ago.

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